Optimization Techniques – Group Assignment

Integer Programming for Inventory Cost Management

By:

Sanjeev Sidharth, Amol Pol, Anu Rathi, Jai Kushwaha

**Problem statement & case**:

*In the next six months a company must, in each month, operate either a normal shift or an extended shift (if it produces at all). A normal shift costs 100,000 per month and can produce up to 5,000 units per month. An extended shift costs 180,000 per month and can produce up to 7,500 units per month.*

*It is estimated that changing from a normal shift in one month to an extended shift next month costs an extra £15,000. No extra cost is incurred in changing from an extended shift in one month to a normal shift in the next month.*

*Carrying cost is estimated to be 2 per unit per month (based on the stock held at the end of each month) and the initial stock is 3,000 units (produced by a normal shift). At the end of month 6, at least 2000 units should be in stock. The demand for the company's product is shown below:*

*Month 1 2 3 4 5 6*

*Demand 6,000 6,500 7,500 7,000 6,000 6,000*

*IF the company produces anything in a particular month it must produce at least 2,000 units.If the company wants a production plan for the next six months that avoids stockouts, formulate their problem as an integer program.*

**Decision Variables** -

N1 (Integer) = Number of units produced with Normal shift in month 1

N2 (Integer) = Number of units produced with Normal shift in month 2

N3 (Integer) = Number of units produced with Normal shift in month 3

N4 (Integer) = Number of units produced with Normal shift in month 4

N5 (Integer) = Number of units produced with Normal shift in month 5

N6 (Integer) = Number of units produced with Normal shift in month 6

E1 (Integer) = Number of units produced with Extended shift in month 1

E2 (Integer) = Number of units produced with Extended shift in month 2

E3 (Integer) = Number of units produced with Extended shift in month 3

E4 (Integer) = Number of units produced with Extended shift in month 4

E5 (Integer) = Number of units produced with Extended shift in month 5

E6 (Integer) = Number of units produced with Extended shift in month 6

L1 (Integer) = Number of leftover units after month 1

L2 (Integer) = Number of leftover units after month 2

L3 (Integer) = Number of leftover units after month 3

L4 (Integer) = Number of leftover units after month 4

L5 (Integer) = Number of leftover units after month 5

L6 (Integer) = Number of leftover units after month 6

NS1 (Binary) -> 0 = Operation not in Normal Shift in month 1

NS1 (Binary) -> 1 = Operation in Normal Shift in month 1

NS2 (Binary) -> 0 = Operation not in Normal Shift in month 2

NS2 (Binary) -> 1 = Operation in Normal Shift in month 2

NS3 (Binary) -> 0 = Operation not in Normal Shift in month 3

NS3 (Binary) -> 1 = Operation in Normal Shift in month 3

NS4 (Binary) -> 0 = Operation not in Normal Shift in month 4

NS4 (Binary) -> 1 = Operation in Normal Shift in month 4

NS5 (Binary) -> 0 = Operation not in Normal Shift in month 5

NS5 (Binary) -> 1 = Operation in Normal Shift in month 5

NS6 (Binary) -> 0 = Operation not in Normal Shift in month 6

NS6 (Binary) -> 1 = Operation in Normal Shift in month 6

ES1 (Binary) -> 0 = Operation not in Extended Shift in month 1

ES1 (Binary) -> 1 = Operation in Extended Shift in month 1

ES2 (Binary) -> 0 = Operation not in Extended Shift in month 2

ES2 (Binary) -> 1 = Operation in Extended Shift in month 2

ES3 (Binary) -> 0 = Operation not in Extended Shift in month 3

ES3 (Binary) -> 1 = Operation in Extended Shift in month 3

ES4 (Binary) -> 0 = Operation not in Extended Shift in month 4

ES4 (Binary) -> 1 = Operation in Extended Shift in month 4

ES5 (Binary) -> 0 = Operation not in Extended Shift in month 5

ES5 (Binary) -> 1 = Operation in Extended Shift in month 5

ES6 (Binary) -> 0 = Operation not in Extended Shift in month 6

ES6 (Binary) -> 1 = Operation in Extended Shift in month 6

SNE1 (Binary) -> 0 = Operation not in Extended Shift switched from a previous Normal Shift in month 1

SNE1 (Binary) -> 1 = Operation in Extended Shift switched from a previous Normal Shift in month 1

SNE2 (Binary) -> 0 = Operation not in Extended Shift switched from a previous Normal Shift in month 2

SNE2 (Binary) -> 1 = Operation in Extended Shift switched from a previous Normal Shift in month 2

SNE3 (Binary) -> 0 = Operation not in Extended Shift switched from a previous Normal Shift in month 3

SNE3 (Binary) -> 1 = Operation in Extended Shift switched from a previous Normal Shift in month 3

SNE4 (Binary) -> 0 = Operation not in Extended Shift switched from a previous Normal Shift in month 4

SNE4 (Binary) -> 1 = Operation in Extended Shift switched from a previous Normal Shift in month 4

SNE5 (Binary) -> 0 = Operation not in Extended Shift switched from a previous Normal Shift in month 5

SNE5 (Binary) -> 1 = Operation in Extended Shift switched from a previous Normal Shift in month 5

SNE6 (Binary) -> 0 = Operation not in Extended Shift switched from a previous Normal Shift in month 6

SNE6 (Binary) -> 1 = Operation in Extended Shift switched from a previous Normal Shift in month 6

**Objective Function** –

Our objective here is to minimize the cost while making sure that all constraints are being satisfied.

There are various components which make up the overall cost that the company has to incur.

* $100000 for each month the company operated in Normal shift i.e. 100000(NS1+NS2+NS3+NS4+NS5+NS6)
* $180000 for each month the company operated in Extended shift i.e.

180000(NS1+NS2+NS3+NS4+NS5+NS6)

* $2 per month for each unit carried forward (leftovers) i.e.

2(L1+L2+L3+L4+L5+L6)

* $15000 for each month where operation was in extended shift switched from a normal shift in previous month. i.e.

15000(SNE1+ SNE2+ SNE3+ SNE4+ SNE5+ SNE6)

Adding them all up we get our objective function of Z and our aim is to minimize it (using solver).

Min Z = 100000NS1 + 100000NS2 + 100000NS3 + 100000NS4 + 100000NS5 + 100000NS6 +

180000ES1 + 180000ES2 + 180000ES3 + 180000ES4 + 180000ES5 + 180000ES6 +

2L1 + 2L2 + 2L3 + 2L4 + 2L5 + 2L6 +

15000SNE1 + 15000SNE2 + 15000SNE3 + 15000SNE4 + 15000SNE5 + 15000SNE6

**Constraint set 1**-

For -> Monthly Demand (Refer table in question)

For -> 3000 units available at the start of Month 1

For -> 2000 units should be available at the end of Month 6

N1 + E1 + 3000 – 6000 = L1 ->

N1 + E1 – L1 = 3000 **[1]**

N2 + E2 + L1 – 6500 = L2 ->

N2 + E2 + L1 – L2 = 6500 **[2]**

N3 + E3 + L2 – 7500 = L3 ->

N3 + E3 + L2 – L3 = 7500 **[3]**

N4 + E4 + L3 – 7000 = L4 ->

N4 + E4 + L3 – L4 = 7000 **[4]**

N5 + E5 + L4 – 6000 = L5 ->

N5 + E5 + L4 – L5 = 6000 **[5]**

N6 + E6 + L5 – 6000 = L6 ->

N6 + E6 + L5 – L6 = 6000 **[6]**

L6 >= 2000 **[7]**

**Constraint set 2**-

For -> Maximum units that can be produced in Normal shift is 5000

For -> Maximum units that can be produced in Extended shift is 7500

For -> in any particular month, if a shift is normal, no. of units generated in extended shift is 0 and vice-versa.

N1 - 5000NS1 <= 0 **[8]**

N2 - 5000NS2 <= 0 **[9]**

N3 - 5000NS3 <= 0 **[10]**

N4 - 5000NS4 <= 0 **[11]**

N5 - 5000NS5 <= 0 **[12]**

N6 - 5000NS6 <= 0 **[13]**

E1 – 7500ES1 <= 0 **[14]**

E2 – 7500ES2 <= 0 **[15]**

E3 – 7500ES3 <= 0 **[16]**

E4 – 7500ES4 <= 0 **[17]**

E5 – 7500ES5 <= 0 **[18]**

E6 – 7500ES6 <= 0 **[19]**

**Constraint set 3**-

For -> At least 2000 units should be produced whenever there is production in any particular month

Extra Binary Variables -> A1:A6 (for N1:N6) and B1:B6 (for E1:E6) taken to convert OR conditions to AND conditions

N1 = 0

OR

N1 >= 2000

Can be written as a function of A1 (to convert to AND conditions)

N1 <= 5000A1 ->

N1 - 5000A1 <= 0

AND

2000 – N1 <= 5000(1 – A1) ->

5000A1 – N1 <= 3000

Similarly all other constraint equations can be formed.

Writing the whole set together below –

N1 - 5000A1 <= 0 **[20]**

5000A1 – N1 <= 3000 **[21]**

N2 - 5000A2 <= 0 **[22]**

5000A2 – N2 <= 3000 **[23]**

N3 - 5000A3<= 0 **[24]**

5000A3 – N3 <= 3000 **[25]**

N4 - 5000A4 <= 0 **[26]**

5000A4 – N4 <= 3000 **[27]**

N5 - 5000A5 <= 0 **[28]**

5000A5 – N5 <= 3000 **[29]**

N6 - 5000A6 <= 0 **[30]**

5000A6 – N6 <= 3000 **[31]**

E1 - 7500B1 <= 0 **[32]**

7500B1 – E1 <= 5500 **[33]**

E2 - 7500B2 <= 0 **[34]**

7500B2 – E2 <= 5500 **[35]**

E3 - 7500B3<= 0 **[36]**

7500B3 – E3 <= 5500 **[37]**

E4 - 7500B4 <= 0 **[38]**

7500B4 – E4 <= 5500 **[39]**

E5 - 7500B5 <= 0 **[40]**

7500B5 – E5 <= 5500 **[41]**

E6 - 7500B6 <= 0 **[42]**

7500B6 – E6 <= 5500 **[43]**

**Constraint set 4**-

For -> changing from a normal shift in one month to an extended shift next month costs an extra 15,000

For -> Initial stock was produced by a Normal Shift in previous month

Extra Binary Variables -> C2:C6 (for SNE2:SNE6) taken to convert IF-THEN conditions to AND conditions

SNE1 = ES1

SNE1 - ES1 = 0 **[44]**

Further

IF ES2 + NS1 > 1

THEN SNE2 > 1

Can be written as a function of C2 (to convert to AND conditions)

ES2 + NS1 – 1 > 0 ->

ES2 + NS1 – 1 <= 100(1-C2) ->

ES2 + NS1 + 100C2 <= 101

AND

SNE2 – 1 >= 0 ->

1 – SNE2 <= 100C2 ->

100C2 + SNE2 >= 1

Similarly all other constraint equations can be formed.

Writing the whole set together below –

ES2 + NS1 + 100C2 <= 101 **[45]**

100C2 + SNE2 >= 1 **[46]**

ES3 + NS2 + 100C3 <= 101 **[47]**

100C3 + SNE3 >= 1 **[48]**

ES4 + NS3 + 100C4 <= 101 **[49]**

100C4 + SNE4 >= 1 **[50]**

ES5 + NS4 + 100C5 <= 101 **[51]**

100C5 + SNE5 >= 1 **[52]**

ES6+ NS5 + 100C6 <= 101 **[53]**

100C6 + SNE6 >= 1 **[54]**

**Constraint set 5**-

For -> in any particular month, shift can either be Normal, or Extended.

NS1 + ES1 = 1 **[55]**

NS1 + ES1 = 1 **[56]**

NS1 + ES1 = 1 **[57]**

NS1 + ES1 = 1 **[58]**

NS1 + ES1 = 1 **[59]**

NS1 + ES1 = 1 **[60]**

**Observations** (In Result from Solver) –

* The minimum amount required to meet all the constraints is $**943000**
* The company must operate in **Normal shifts** for Month 1 and Month 2, producing 4500 and 5000 units respectively.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| N1 | N2 | N3 | N4 | N5 | N6 |
| 4500 | 5000 | 0 | 0 | 0 | 0 |

* The company must switch to extended shifts in Month 3 and continue to work in **extended shifts** till Month 6, producing 7500, 7000, 6500 and 7500 units respectively.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E4 | E5 | E6 |
| 0 | 0 | 7500 | 7000 | 6500 | 7500 |

* Leftovers for Month 1, 5 and 6 are 1500, 500 and 2000 respectively for optimized model.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| L1 | L2 | L3 | L4 | L5 | L6 |
| 1500 | 0 | 0 | 0 | 500 | 2000 |

* Normal shift is required for Month1 and Month2 for the optimized production level,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| NS1 | NS2 | NS3 | NS4 | NS5 | NS6 |
| 1 | 1 | 0 | 0 | 0 | 0 |

* Extended shift is required for the remaining month for maintaining the supply and demand for the company i.e. for months 3, 4, 5 and 6

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ES1 | ES2 | ES3 | ES4 | ES5 | ES6 |
| 0 | 0 | 1 | 1 | 1 | 1 |

* Operation in Extended Shift switched from a previous Normal Shift in month 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| SNE1 | SNE2 | SNE3 | SNE4 | SNE5 | SNE6 |
| 0 | 0 | 1 | 0 | 0 | 0 |